Transient Analysis of Tapered Transmission Lines Used as Transformers for Short Pulses

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Abstract—**The transient behavior of tapered transmission lines is studied in detail by investigating their step responses by an improved method of characteristics. We take interest in the first arriving wave and following dropping process at the load end which play important roles in determining the response waveform and power coupling efficiency under short pulse excitation. Numerical results show that, for given load and source impedances and propagation delay, the magnitude of the first arriving wave is invariable for any tapered line under both ends are well matched, and the slowest dropping is reached as the characteristic impedance distribution satisfies some condition. The concept of instantaneous dropping speed is used in further theoretical analysis and the numerical results are verified by theoretical formulas. Finally, we show the relation between the instantaneous dropping speed and frequency-domain characteristics.**

I. Introduction

In MICROWAVE engineering, tapered or nonuniform transmission lines are widely used as impedance transformers, impedance matching sections, filters, resonators, etc. With the developing of high-speed pulse technique, pulses with duration in picosecond range have become commonplace in modern digital systems. Frequency analysis shows that most of the power of these short pulses lies in the waveband of microwave and thus it is possible to apply the tapered lines as pulse transformers in improving impedance matching [1]–[3].

Many authors have contributed to the study of tapered lines [4]–[11]. However, most previous studies are limited in frequency domain or in numerical simulation, thus in most cases the time-domain characteristics of tapered lines were not studied and depicted directly. We would like to mention particularly that Hsue and Hechtman [9] used a multiple-section approximating method to investigate the step response of tapered lines and concluded that the exponential line provided the maximum first arriving wave followed by a decayed transition ripple to the load end and thus had potential application in pulse transformers and pulse waveform alteration.

In this paper, however, it is found by us that all tapered lines can reach this maximum first arriving wave. We use an improved method of characteristics to investigate the step response of tapered lines. Both numerical calculations and theoretical analysis are carried out by this new method. The step response is similarly divided into three parts as in [9]: (1) the first arriving wave, (2) the dropping process, and (3) the steady-state region. We mainly take interest in the first arriving wave and following dropping process which play important roles in determining the response waveform and power coupling efficiency under short pulse excitation. Numerical results show that, under the source and load ends are both matched, for given source and load impedances, the magnitude of the first arriving wave is invariable for all kinds of tapered lines, thus we conclude that the advantage of the exponential lines does not lies in its maximum first arriving wave as concluded by Hsue and Hechtman [9]. Therefore, we pay more attention to the dropping process and use the concept of fall time to depict the time-domain characteristics of tapered lines. Under matched condition, the numerical results also show that, for given source and load impedances and propagation delay, the exponential line (in electrical length) has the slowest dropping speed and thus the maximum pulse power coupling efficiency and the larger the width of the exciting pulse is (as compared with the fall time), the lower the power coupling efficiency is. The behavior of a tapered line approaches the behavior of an ideal transformer as its fall time is larger than the width of the excitation pulse and with the load end mismatched, the behavior worsens. Instantaneous dropping speed at the start of the response is used in further theoretical analysis and theoretical formulas are derived for calculating the fall time and the numerical results are verified by them. Finally, the relation between the instantaneous dropping speed and corresponding frequency characteristics is revealed.

II. Theory

Transmission lines (under quasi-TEM mode approximation) are governed by the following standard equations [7], [8]

$$\frac{\partial V}{\partial x_1} + L\frac{\partial I}{\partial t_1} + RI = 0 \tag{1a}$$

$$C\frac{\partial V}{\partial t_1} + \frac{\partial I}{\partial x_1} + GV = 0 \tag{1b}$$

where $C(x_1)$, $L(x_1)$, $R(x_1)$, $G(x_1)$ are the capacitance, inductance, resistance, and conductance per unit length, respectively. The propagation speed and characteristic impedance

Manuscript received November 30, 1994; revised August 1, 1995. This work is supported by Chinese Aeronautic Scientific Foundation.

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IEEE Log Number 9414839.

are, respectively, given by

$$P_S(x_1) = 1/\sqrt{L(x_1)C(x_1)} = c/\sqrt{\epsilon_{\text{eff}}(x_1)}$$

$$Z(x_1) = \sqrt{L(x_1)/C(x_1)}$$

where $\epsilon_{\text{eff}}(x_1)$ is the effective permittivity and c denotes velocity of light in free space. In this paper, we deal with tapered line which $Z(x_1)$ varies continuously along the x_1 direction. As a matching section for short pulses, the tapered line itself is short in length and the influence of $R(x_1)$ and $G(x_1)$ is negligible. Also, for the convenience of theoretical analysis, it is assumed that the propagation speed as well as the characteristic impedance is independent of frequency, which means that no dispersion is considered. This is an effective approximation especially under the excitation pulse is relatively wide and the length of the line is short enough. Therefore, equation (1a) and (1b) become

$$P_s\frac{\partial V}{\partial x_1} + Z\frac{\partial I}{\partial t_1} = 0 \tag{2a}$$

$$\frac{\partial V}{\partial t_1} + P_s Z\frac{\partial I}{\partial x_1} = 0. \tag{2b}$$

We use the following transformations as used in [8]

$$p = \frac{V}{\sqrt{Z}} + I\sqrt{Z} \qquad q = \frac{V}{\sqrt{Z}} - I\sqrt{Z}. \tag{3}$$

Also, assuming the physical length of the tapered line is l, normalized spatial and temporal coordinates are introduced by

$$x = \int_0^{x_1} \frac{dx_1}{P_s(x_1)} \Big/ T_d \qquad t = t_1/T_d \tag{4}$$

where

$$T_d = \int_0^l \frac{dx_1}{P_s(x_1)}$$

is the propagation delay time. It is to be noted that the x coordinate is proportional to the so-called electrical length. Then equations (2a) and (2b) become

$$\frac{\partial q}{\partial t} - \frac{\partial q}{\partial x} = \frac{1}{2}\frac{d(\ln Z)}{dx}p \tag{5a}$$

$$\frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} = -\frac{1}{2}\frac{d(\ln Z)}{dx}q. \tag{5b}$$

In new coordinates, the length and propagation delay time are both 1.0 and the solution space is defined by $0 \le t < \infty$, $0 \le x \le 1$.

Equations (5a) and (5b) are of the hyperbolic type and the characteristic lines are $t - x =$ constant and $t + x =$ constant. The characteristic lines suggest new coordinates as $\xi = t + x$, $\eta = t - x$ so that $\xi =$ constant, $\eta =$ constant are the characteristic lines. We transform (5a) and (5b) to ξ, η coordinates to get

$$\frac{\partial q}{\partial \eta} = -\frac{1}{2}\frac{\partial(\ln Z)}{\partial \eta}p \tag{6a}$$

$$\frac{\partial p}{\partial \xi} = -\frac{1}{2}\frac{\partial(\ln Z)}{\partial \xi}q. \tag{6b}$$

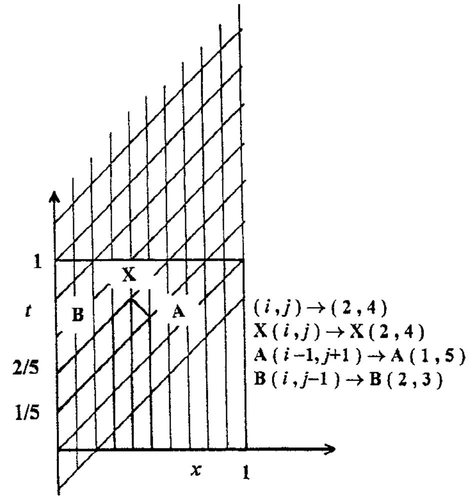

Fig. 1. A new discretization scheme ($N = 10$, $\Delta x = 1/10$, $\Delta t = 1/5$).

From (6a) and (6b), (5a) and (5b) can be numerically solved by the method of characteristics as used in [7]. However, here we apply the method of characteristic with a new discretization scheme which is illustrated in Fig. 1.

Letting $\Delta x = 1/N$ and $\Delta t = 2/N$, where N is an integer, we consider the lines $t - x = i\,\Delta t$ $(i = -1, 0, \ldots)$ and $x = j\,\Delta x$ $(j = 0, 1, \ldots, N)$. The point of intersection of $t - x = i\,\Delta t$ and $x = j\,\Delta x$ is designated as (i, j). Then, for $i > -1, 0 < j < N$, the point $X(i, j)$ is connected to the point $A(i-1, j+1)$ through the characteristic line $\xi = (i + j)\,\Delta t$ and to the point $B(i, j-1)$ through the characteristic line $\eta = i\,\Delta t$, respectively. Equation (6a) is integrated along line $\xi = (i + j)\,\Delta t$ from A to X and (6b) is integrated along line $\eta = i\,\Delta t$ from B to X, respectively. Then, we get the following

$$q_i^j - q_{i-1}^{j+1} = -\frac{1}{2}\int_{\eta_A}^{\eta_X} p\frac{\partial(\ln Z)}{\partial \eta}\,d\eta \tag{7a}$$

$$p_i^j - p_i^{j-1} = -\frac{1}{2}\int_{\xi_B}^{\xi_X} q\frac{\partial(\ln Z)}{\partial \xi}\,d\xi \tag{7b}$$

where q_i^j, p_i^j stand for the value of p, q at (i, j). The trapezoidal rule

$$\int_x^{x+\Delta x} \frac{df(x)}{dx}g(x)\,dx = \frac{1}{2}[g(x) + g(x+\Delta x)] \cdot [f(x+\Delta x) - f(x)] + o(\Delta x^2)$$

is used to evaluate the integrals so that (7a) and (7b) become

$$q_i^j = q_{i-1}^{j+1} - \frac{1}{4}(p_i^j + p_{i-1}^{j+1})\ln\frac{Z(j\,\Delta x)}{Z((j+1)\,\Delta x)} + o(\Delta x^2) \tag{8a}$$

$$p_i^j = p_i^{j-1} - \frac{1}{4}(q_i^j + q_i^{j-1})\ln\frac{Z(j\,\Delta x)}{Z((j-1)\,\Delta x)} + o(\Delta x^2) \tag{8b}$$

where $o(\Delta x^2)$ represents a higher order infinitesimal than Δx^2 as $\Delta x \to 0$.

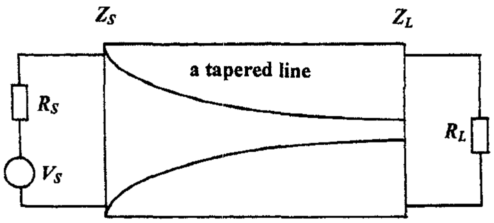

Fig. 2. A tapered line used as a matching section.

For a sufficiently large value of N, the higher order infinitesimal terms are negligible. Assuming that the value of p, q is known at A and B, we can evaluate p, q at X by using (8a) and (8b). However, at the two ends of the tapered line which implies $j = 0$ or $j = N$, one of (8a) and (8b) must be discarded and replaced by corresponding boundary condition equation. In the special case of zero initial values of $V(x_1, t_1)$ and $I(x_1, t_1)$, both $p(t, x)$ and $q(t, x)$ are zero until $t \geq x$ [7]. Thus we have

$$p_{-1}^j = 0,\ q_{-1}^j = 0 (j = 0, 1, \cdots, N). \tag{9}$$

Then, by using (8a), (8b), and (9) and the boundary conditions, we can evaluate

$$p_0^j,\ q_0^j (j = 0, 1, \cdots, N)$$

and so on and so forth we can evaluate

$$p_i^j,\ q_i^j (i = 0, 1, \cdots; j = 0, 1, \cdots, N).$$

Practical calculations indicate that, as an algorithm, this new discrezation scheme makes numerical calculations more efficient than the conventional scheme as used in [7]. Further theoretical analysis is also based on it.

III. Numerical Results

Tapered line as a pulse transformer between the source and load is illustrated in Fig. 2. R_S and R_L denote the source and load impedances. Z_S and Z_L denote the characteristic impedance of the tapered line at the source and load ends. V_S is the voltage source signal. Only the special case of R_S and R_L being purely resistive is considered in this paper.

In Fig. 2, the boundary conditions at the source and load ends of the tapered line are $V + IR_S = V_S$ and $V - R_L I = 0$, respectively. Letting $a = (Z_S - R_S)/(Z_S + R_S)$ and $b = (Z_L - R_L)/(Z_L + R_L)$, by using (3), the boundary conditions become

$$p + aq = (1 + a)\frac{V_S}{\sqrt{Z_S}} \tag{10a}$$

$$bp + q = 0. \tag{10b}$$

Also by using (3), the unit delayed output waveform V_L at the load end is

$$V_L(1 + i\,\Delta t) = \frac{1}{2}(p_i^N + q_i^N)\sqrt{Z_L}. \tag{11}$$

In Fig. 3, we show the numerical unit step responses for different $Z(x)$ distributions under $R_S = 50\Omega$, $R_L = 5\Omega$, $a = 0$, $b = 0$. The condition of $a = 0$ and $b = 0$ means that the source and load ends are well matched.

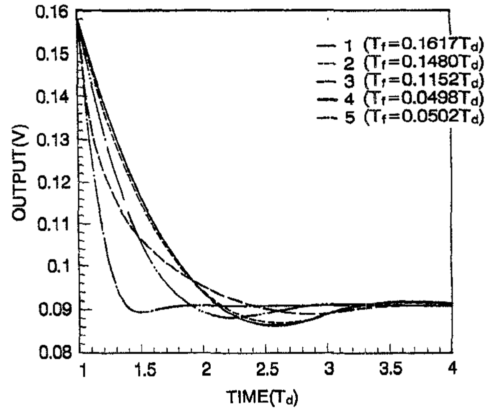

Fig. 3. The unit step response waveforms under $R_S = 50\Omega$, $R_L = 5\Omega$, $a = 0$, $b = 0$. 1: Exponential line ($Z(x) = 50.0\exp(-2.3026x)$), 2: Bessel line 1 ($Z(x) = 10.69(x + 0.4625)^{-2}$), 3: parabolic line ($Z(x) = 50.0 - 135.0x^2 + 90.0x^3$), 4: Bessel line 2 ($Z(x) = 5.025(x + 0.0101)^{-1/2}$), 5: hyperbolic line ($Z(x) = 27.5 - 22.5\tanh(10.0 - 5.0x)$).

The numerical results show that, for all tapered lines, the magnitude of the first arriving voltage reaches the same magnitude as ideal transformer is used, which is $(5/50)^{1/2}/2 \approx 0.158$, while the dropping speed varies for different lines. The duration for V_L to decrease to 90% of the first arriving voltage is defined as fall time which is designated as T_f. The values of T_f are also given in Fig. 3.

T_f is an important parameter concerning the time domain characteristics of the tapered lines. It is predicated that, as a pulse transformer, the tapered line is effective only under the excitation is short pulses with duration smaller or not very larger than T_f and a larger T_f is always expected for better pulse power coupling efficiency. The behavior of a tapered line approaches the behavior of an ideal transformer as its fall time is larger than the width of the excitation pulse. So far as our numerical results show, for a given T_d, T_f gets the maximum value as $Z(x)$ is an exponential distribution. Thus, we conclude that the advantage of the exponential line (in electrical length) lies in its maximum fall time instead of its maximum first arriving wave as predicted in [9]. By increasing T_d, we can also increase T_f, which means increasing l or decreasing P_S of the tapered line. In [2], high-dielectric constant material is used to get a lower propagation speed to decrease the total length of the TLT.

In Fig. 4, we show the corresponding numerical responses under a Gaussian pulse excitation. The width (at the half magnitude points) and peak voltage of the Gaussian pulse is, respectively, $0.25T_d$ and 1 V. We can see that, the width of the response pulses is almost equal to the width of the excitation pulse and the peak voltages eaches the maximum as $Z(x)$ is an exponential distribution. In Fig. 5, we show the two curves of output pulse peak voltage and pulse width ratio of output to input as the width of the exciting Gaussian pulse changes from $0.3T_f$ to $18T_f$, under $R_S = 50\Omega$, $R_L = 5\Omega$, $a = 0$, $b = 0$ and $Z(x)$ is an exponential distribution. It is easy to see that, as the width of the excitation pulse approaches to $18T_f$, the tapered line is more and more ineffective.

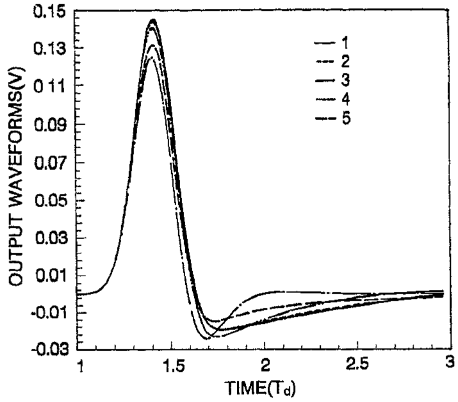

Fig. 4. Corresponding response waveforms under a Gaussian pulse excitation.

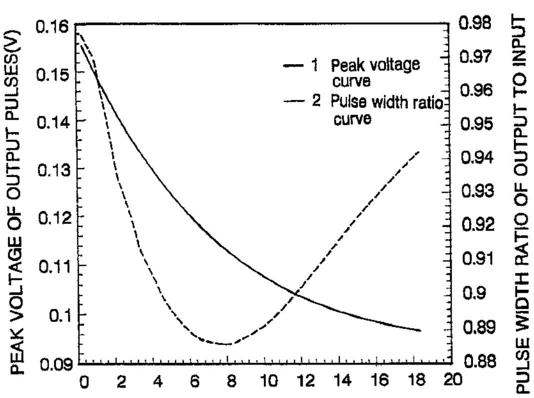

Fig. 5. Output pulse peak voltage and pulse width ratio of output to input as the width of the excitation Gaussian pulse changes.

In some cases, for example, if the load is a low impedance laser diode, it is very difficult for Z_L to be equal to R_L [3]. This is equivalent to $b \neq 0$. In Fig. 4, we show the numerical unit step responses for $b \neq 0$, assuming $Z(x)$ is an exponential distribution and $R_S = 50\Omega$, $R_L = 5\Omega$, $a = 0$.

We can see that, as $|b|$ increases, the magnitude of the first arriving voltage decreases and the dropping speed changes. With the load end mismatched, the behavior of the tapered line as a pulse transformer worsens.

IV. Further Theoretical Investigation

In this section, theoretical analysis is carried out to verify the preceding numerical results.

Assuming M is the magnitude of the step source and $a = 0$, we consider (7a) and (7b) together with the boundary conditions (10a) and (10b). By using the following rule

$$\int_x^{x+\Delta x} \frac{df(x)}{dx} g(x)\,dx = g(x)[f(x+\Delta x) - f(x)] + o(\Delta x)$$

to evaluate the integral in (7a) and the trapezoidal rule to evaluate the integral in (7b), we have

$$q_i^j = \begin{cases} q_{i-1}^{j+1} - \frac{1}{2} p_{i-1}^{j+1} \ln \frac{Z(j\,\Delta x)}{Z((j+1)\,\Delta x)} + o(\Delta x) \\ \quad \text{for } j = 0, 1, \cdots, N-1 \\ -b p_i^j \\ \quad \text{for } j = N \end{cases} \tag{12a}$$

$$p_i^j = \begin{cases} M/\sqrt{Z_S} \\ \quad \text{for } j = 0 \\ p_i^{j-1} - \frac{1}{4}(q_i^j + q_i^{j-1}) \ln \frac{Z(j\,\Delta x)}{Z((j-1)\,\Delta x)} + o(\Delta x^2) \\ \quad \text{for } j = 1, 2, \cdots, N. \end{cases} \tag{12b}$$

Let $i = 0$, making use of (9) and taking the limit $\Delta x \to 0$, we can easily get

$$q_0^j = \begin{cases} 0 \quad \text{for } j = 0, 1, \cdots, N-1 \\ -bM/\sqrt{Z_S} \quad \text{for } j = N \end{cases} \tag{13a}$$

$$p_0^j = M/\sqrt{Z_S} \quad \text{for } j = 0, 1, \cdots, N. \tag{13b}$$

By using (11), we get

$$V_L(1.0) = \frac{1}{2}(p_0^N + q_0^N)\sqrt{Z_L} = \frac{1}{2} M \sqrt{1-b^2} \sqrt{\frac{R_L}{R_S}}. \tag{14}$$

In case of $b = 0$, it is the very result as Hsue and Hechtman [9] have obtained in the special case of exponential lines. The above formula shows that the first arriving voltage is independent of $Z(x)$ distribution and it reduces as $b \neq 0$.

Now, substituting $i = 1$ into (12a) and (12b), by using (13a) and (13b) we obtain

$$q_1^j = \begin{cases} -\frac{1}{2}\frac{M}{\sqrt{Z_S}} \ln \frac{Z(j\,\Delta x)}{Z((j+1)\,\Delta x)} + o(\Delta x) \\ \quad \text{for } j = 0, 1, \cdots, N-2 \\ -b\frac{M}{\sqrt{Z_S}} - \frac{1}{2}\frac{M}{\sqrt{Z_S}} \ln \frac{Z(j\,\Delta x)}{Z((j+1)\,\Delta x)} + o(\Delta x) \\ \quad \text{for } j = N-1 \\ -b p_1^j \\ \quad \text{for } j = N \end{cases} \tag{15a}$$

$$p_1^j = \begin{cases} M/\sqrt{Z_S} \\ \quad \text{for } j = 0 \\ p_1^{j-1} - \frac{1}{4}(q_1^j + q_1^{j-1}) \ln \frac{Z(j\,\Delta x)}{Z((j-1)\,\Delta x)} + o(\Delta x^2) \\ \quad \text{for } j = 1, 2, \cdots, N. \end{cases} \tag{15b}$$

By substituting (15a) into (15b) and using the following formula

$$\ln \frac{f(x+\Delta x)}{f(x)} = \frac{df(x)}{dx}\frac{1}{f(x)}\Delta x + o(\Delta x)$$

after some manipulations on infinitesimal we obtain

$$p_1^N = \frac{M}{\sqrt{Z_S}} - \frac{M}{\sqrt{Z_S}} \sum_{j=1}^{N-1} r^2(j\,\Delta x)\,\Delta x^2 + \frac{3b}{2}\frac{M}{\sqrt{Z_S}} r(1.0)\,\Delta x + o(\Delta x)$$

where

$$r(x) = \frac{dZ(x)}{dx}\frac{1}{2Z(x)}$$

is the differential reflection coefficient distribution [5]. By using (11), we have

$$\begin{aligned} V_L(1+\Delta t) &= \frac{1}{2}\sqrt{Z_L}(1-b)\left[\frac{M}{\sqrt{Z_S}} - \frac{M}{\sqrt{Z_S}}\sum_{j=1}^{N-1} r^2(j\,\Delta x)\,\Delta x^2 \right. \\ &\quad \left. + \frac{3b}{2}\frac{M}{\sqrt{Z_S}} r(1.0)\,\Delta x + o(\Delta x)\right]. \end{aligned} \tag{16}$$

The dropping speed is approximately described by the instantaneous dropping speed at $t = 1$ which is

$$D_f = \lim_{\Delta t \to 0}\left|\frac{V_L(1+\Delta t) - V_L(1.0)}{\Delta t}\frac{1}{V_L(1.0)}\right|. \tag{17}$$

By substituting (14) and (16) into (17) and noting the higher order infinitesimal is negligible and $\Delta t = 2\,\Delta x$, we obtain

$$D_f = \left|\frac{3b}{2} r(1.0) - \frac{1}{2}\int_0^1 r^2(x)\,dx\right|. \tag{18}$$

In the special case of $b = 0$, we get

$$D_f = \frac{1}{2}\int_0^1 r^2(x)\,dx. \tag{19}$$

By applying the following theorem

$$\left(\int_0^1 f(x)g(x)\,dx\right)^2 \le \int_0^1 f^2(x)\,dx \int_0^1 g^2(x)\,dx$$

under $b = 0$, we have

$$D_f = \frac{1}{2}\int_0^1 r^2(x)\int_0^1 1\,dx \ge \frac{1}{2}\left(\int_0^1 r(x)\,dx\right)^2 = \frac{1}{8}\left(\ln\frac{Z_L}{Z_S}\right)^2. \tag{20}$$

The equality holds only if $r(x) =$ constant is satisfied, which means $Z(x)$ is an exponential distribution. By using (4), $Z(x_1)$ is given as following

$$Z(x_1) = c_1 \exp\left(c_2\int_0^{x_1}\frac{dx_1}{P_S(x_1)}\right)$$

where c_1 and c_2 are undetermined constants. Therefore, exponential lines (in electrical length) have the minimum D_f for given Z_S and Z_L.

The numerical results are verified by the above theoretical analysis. Assuming the dropping near $t = 1$ is nearly linear, the following approximate relation is easily obtained

$$T_f \approx 0.1\frac{T_d}{D_f}. \tag{21}$$

In the special case of exponential lines, it turns into

$$T_f \approx \frac{0.8T_d}{\ln^2(Z_L/Z_S)}.$$

Under $Z_S = 50\Omega$, $Z_L = 5\Omega$, we have $T_f \approx 0.151T_d$. The relative error is 6.6% as compared with the numerical result in Fig. 3. The TLT described in [1] is assumed to be an exponential line and the effective permittivity is invariably 10.0 along its length for its unconventional structure. Its total length is 50 mm, so $T_d = 527$ ps. Therefore, under $Z_S = 50\Omega$, $Z_L = 5.5\Omega$, $a = 0$, $b = 0$, we have $T_f \approx 86$ ps.

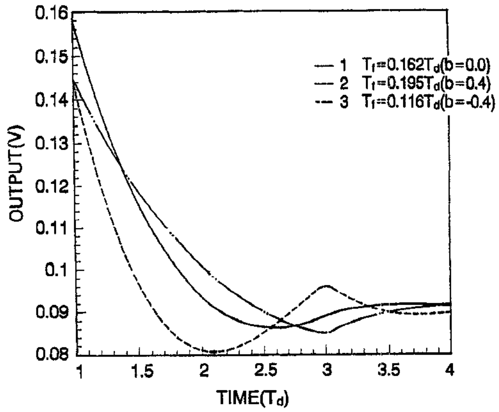

Fig. 6. Step response waveforms for $b \neq 0$.

V. Frequency-Domain Consideration

In microwave, tapered lines are often characterized by the input reflection coefficient as a function of frequency. Under $\rho \ll 1$, $a = 0$, $b = 0$ and the propagation speed P_S is a constant, the following approximate formula is used to calculate the reflection coefficient [4], [5]

$$\rho(f_1) = \int_0^l r(x_1)\exp\left(\frac{-j4\pi f_1 x_1}{P_S}\right)dx_1 \tag{22}$$

where $r(x_1)$ is the differential reflection coefficient distribution and l is the length of the tapered line. By the transformation of $x = x_1/l$ and $f = 2f_1T_d$, it becomes

$$\rho(f) = \int_0^1 r(x)\exp(-j2\pi f x)\,dx.$$

By applying Parseval theorem we obtain

$$\int_{-\infty}^{+\infty}|\rho|^2\,df = \int_0^1 r^2(x)\,dx = 2D_f \tag{23}$$

which show the relation between D_f and the frequency domain characteristics. Under P_S is not a constant, by (3), (2a) and (2b) can always be equivalently transformed into equations with invariable P_S (=1.0). Thus, (23) is still effective.

By (23), the minimum D_f of the exponential line implies that the left of (23) have the minimum value. In Fig. 7, we show the input reflection coefficient curve of different lines under $Z_S = 50\Omega$, $Z_L = 150\Omega$, $a = 0$, $b = 0$, which is calculated by processing the corresponding step response waveforms by FFT. In Fig. 7, the minimum D_f of the exponential line is obvious.

VI. Conclusion

The above detailed analysis of the transient phenomena on tapered lines has shown that tapered lines can be used as pulse transformers for short pulses and the difference between

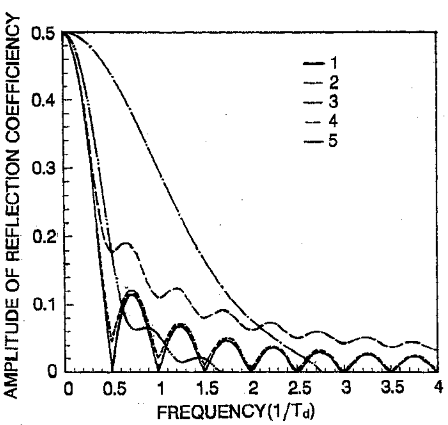

Fig. 7. Input reflection coefficient of different tapered lines under $Z_S = 50\Omega$, $Z_L = 150\Omega$, $a = 0$, $b = 0$. 1: Exponential line ($Z(x) = 50.0 \exp(1.0986x)$), 2: Bessel line 1 ($Z(x) = 26.8(1.366+x)^2$), 3: parabolic line ($Z(x) = 50.0 + 300.0x^2 - 200.0x^3$), 4: Bessel line 2 ($Z(x) = 141.42(x + 0.125)^{1/2}$), 5: hyperbolic line ($Z(x) = 100.0 + 50.0 \tanh(10.0x - 5.0)$).

different lines lies in the dropping speed instead of the first arriving wave. By using the instantaneous dropping speed to describe the dropping behavior, we successfully find a useful formula for calculating the fall time. Both the numerical results and theoretical analysis show that, the exponential line (in electrical length) provides the slowest dropping and thus in view of the power coupling efficiency it is the optimum tapered line pulse transformer. It is to be noted that, dispersion is not considered in this paper and further investigation of the step responses should take account of the effect of dispersion, in which case the performance of a tapered line as a pulse transformer may worsen. Also, the performance of a tapered line worsens if the load end is not completely matched. Finally, we reveal the relation between the instantaneous dropping speed and frequency domain characteristics.

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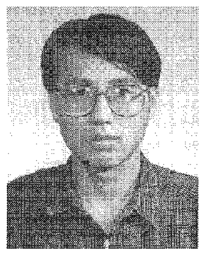

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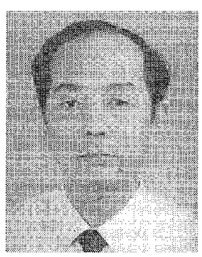

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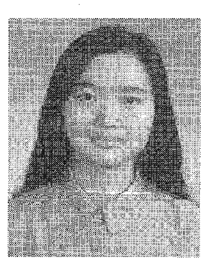

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